Next Generation of Algorithms for Aerodynamic Design Optimization: Current Status and Future Challengers

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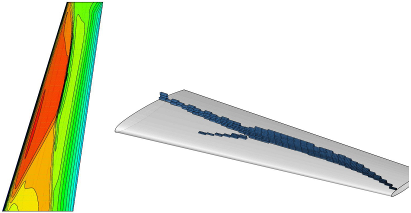
Adjoint-Based Aerodynamic Shape Optimization Using the Drag Decomposition Method (François Bisson)

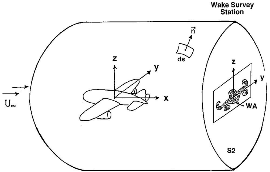
Key Features

- Phenological breakdown of drag
- Reduce mesh dependancy
- Understand the sensitivity of the design variable for each drag component for the full flight envelope
- Potential application to non-planar wings design



DPW-W1 at $M = 0.76$ and $C_L = 0.500$ for fine grid (inviscid)

Mid-Field Drag Decomposition

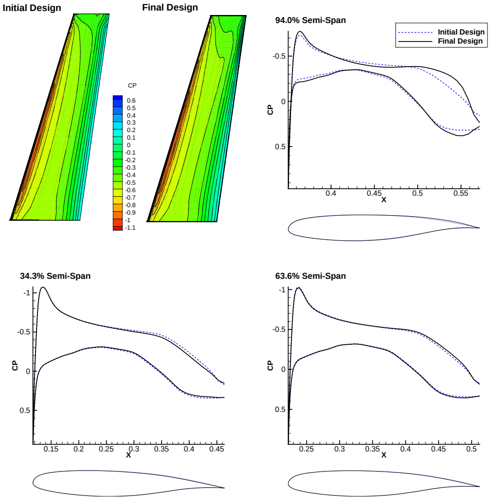


Simplified control volume for the mid-field method (Courtesy [Kusunose et al.,2002])

$$D = \frac{1}{\gamma M_\infty^2} \iint_{S_{Wake}} \left(\frac{\Delta s}{R}\right) \rho \mathbf{u} \cdot \mathbf{n} \, dS + \frac{\rho_\infty}{2} \iint_{S_{Wake}} (\psi\zeta) dS + \mathcal{O}(\Delta^2)$$

- 1^{st} term related to entropy generation processes (shock waves and viscous/artificial dissipations)
- 2^{nd} term is the Maskell's induced drag
  ζ - x-vorticity & ψ - stream function ($\nabla^2 \psi = -\zeta$)

Induced Drag Minimization - DPW-W1 Wing (François Bisson)

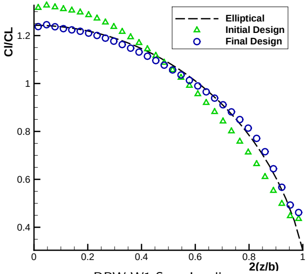


Pressure distribution for DPW-W1 wing induced drag minimization

Fully subsonic ($M = 0.60$)
Lift Constrained ($C_L = 0.40$)
Local surface parametrization

→ Camber optimized to approach elliptical loading

→ 3.43% Induced Drag Reduction

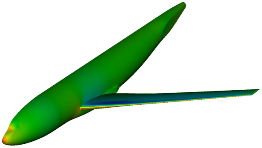


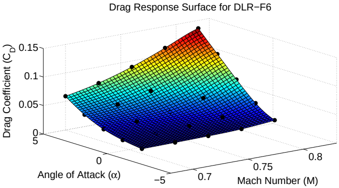
DPW-W1 Span Loading

Sensitivity-Based Sequential Sampling for Surrogate Models (Arthur Paul-Dubois-Taine)

Motivation

- Aircraft design involves a large number of parameters.
- High fidelity CFD results remain expensive at a conceptual design stage.
- Surrogate models → use existing flow solutions to approximate continuous response surface





Question: what criteria do we use to decide on new snapshot locations?

Existing error criteria:

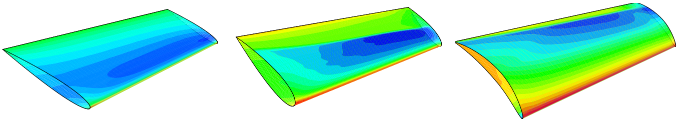
- Mean Square Error (MSE) estimate built in Kriging
- Cross Validation (CV)
  - More accurate than MSE
  - Computationally expensive
  - ⇒ **Objective:** find low cost alternative to CV

New approach: Sensivity analysis ($\mathcal{S}$)

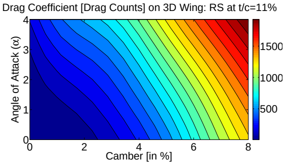
- Uses the mathematical form of Kriging model
- Incorporates gradient information in the sampling process

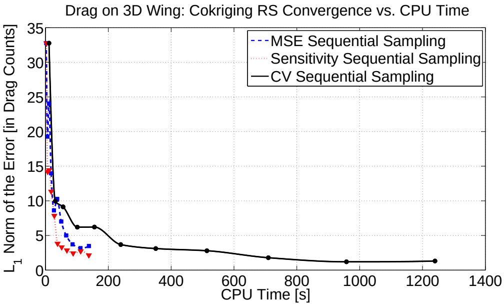
Sensitivity-Based Sequential Sampling for Surrogate Models (Arthur Paul-Dubois-Taine)

Pressure distribution for DPW-W1 at various camber, thickness, and α



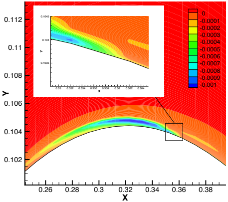
- → Three parameters: Camber, Thickness ratio and Angle of Attack α
- → $M_\infty = .81$, Camber Location $= 40\%$.



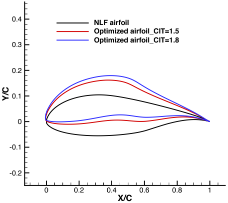


Aerodynamic Optimization of Natural Laminar Flow (NLF) Airfoils

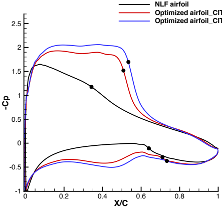
NLF(1)-0416: Minimizing total drag at constant lift, Ma=0.1, Re =2.0 million



Adjoint of Intermittency Factor



Shape modifications



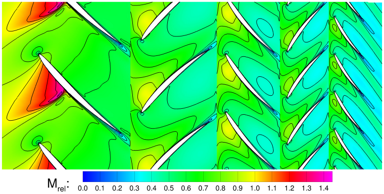
Pressure distribution

- Improvements to $\gamma-\mathrm{Re}_\theta$ model:
  - Khayatzadeh, P and Nadarajah, S, "Laminar-Turbulent Flow Simulation for Wind Turbine Profiles Using the $\gamma-\mathrm{Re}_\theta$ Transition Model", *Wind Energy* (2013), (In-press)
- Developed Adjoint counterpart for $\gamma-\mathrm{Re}_\theta$ model.
  - Khayatzadeh, P and Nadarajah, S, "Aerodynamic Shape Optimization of Natural Laminar Flow (NLF) Airfoils", *50th AIAA Aerospace Sciences Meeting* (2011)

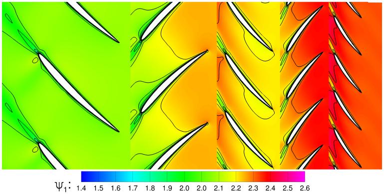
Constrained Aero Optimization for Multistage Turbomachinery

(Benjamin Walther)

2.5-stage transonic compressor. Total pressure ratio: $\pi = 3.0$



M_{rel}, baseline design



First adjoint variable $\psi_1 -$ contours

Walther, B and Nadarajah, S, Constrained Adjoint-Based Aerodynamic Shape Optimization of a Transonic Compressor Stage, *ASME Journal of Turbomachinery* April, 2013.

Walther, B and Nadarajah, S, Constrained Adjoint-Based Aerodynamic Shape Optimization in a Multistage Turbomachinery Environment, *AIAA ASM 2012* January, 2012.

Contributions

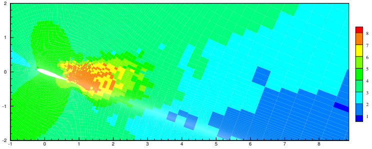
1. Effect of constraint violation on the performance of the compressor stage.
2. *High-lift airfoil design* → reduced number of blades/stages
3. *Unsteady multistage optimization* → rotor-stator interactions using Non-Linear Frequency Domain schemes.

Design Optimization to Alleviate Gust Response and Flutter

(Ali Mosahebi)

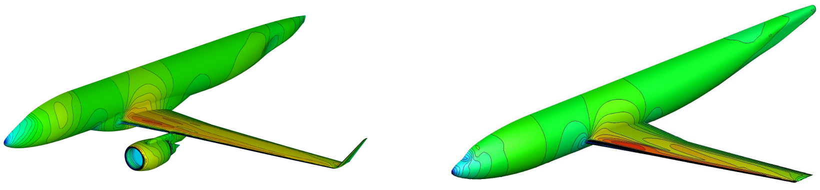
Fast Implicit Adaptive Time Spectral Schemes

$$\left[\frac{1}{\Delta t^*} + \frac{\partial f}{\partial x}\right](x^{n+1,s+1,g+1} - x^{n+1,s+1,g}) = -\left\{\frac{x^{n+1,s+1,g} - x^n}{\Delta t^*} + f^{n+1,s} + \frac{\partial f}{\partial X}\cdot(X^{n+1,s+1,g} - X^{n+1,s})\right\}$$



Kachra, F and Nadarajah, S, Aeroelastic Solutions Using the Non-Linear Frequency Domain Method, *AIAA Journal, 46*(9), September 2008. Mosahebi, A and Nadarajah, S, "An Adaptive NonLinear Frequency Domain Method for Viscous Flows", *Computers and Fluids, Elsevier* (In Press)

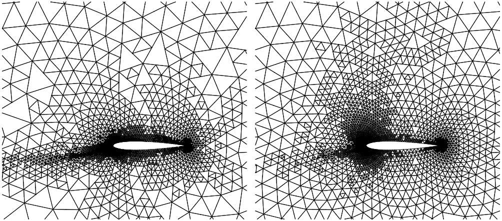
Next-Generation of Algorithms for Aerodynamic Design Optimization



Future

- Potential increase in computing power. Greater parallelism.
- High-order methods.
- Novel approaches to explore the design space. (Multimodality)
- Evaluate sensitivity of design variables on aerodynamic performance for off-design certification cases and include it within the design loop.
- Higher geometric detail during the optimization.
- Quantify uncertainty.
- Evaluating the Hessian.

Current Status of Goal-Oriented Mesh Adaptation (J-S. Cagnone)



Adaptation based on Lift   Adaptation based on Drag

Current State-of-the-Art

- Wide variety of phenomenon / scale lengths
- Reliable CFD requires *a-priori* knowledge of the flow
- Meshing best practices may be insufficient in some cases...

Current Status of Goal-Oriented Mesh Adaptation (J-S. Cagnone)

- Adjoint provides sensitivity of **scalar** quantity of interest
  - Formally $\mathcal{J}(\mathbf{u}) = \int_\Gamma p(\mathbf{u}) ds$
  - In practice, interested in C_l, C_d or C_m
- Can be used to guide mesh refinement
- Theory well understood
  - FEM: Becker & Rannacher (1996,1998)
  - FV: Venditti & Darmofal (2002,2003)
  - DG: Houston & Hartmann (2001,2002)
    Fidkowski & Darmofal (2007)

Some limitations

- What if there is no obvious output?
- What if interested in actual 3D flow field?
- What if interested in a pressure distribution?

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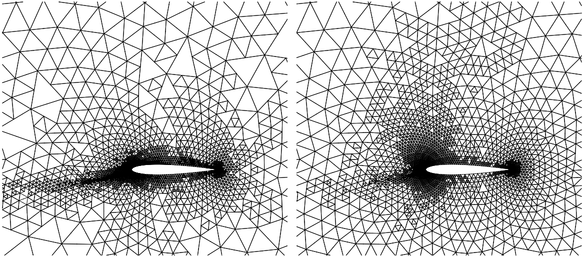

Adaptation based on Lift   Adaptation based on Drag

- Current approaches ask, "*What is the error in the integrated functional?*"

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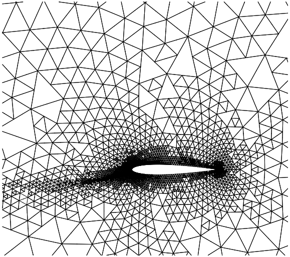
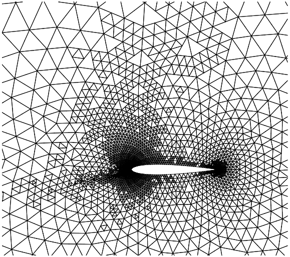


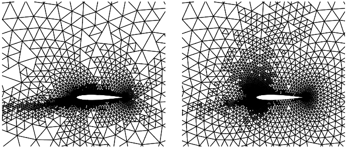
Adaptation based on Lift



Adaptation based on Drag

- Current approaches ask, "*What is the error in the integrated functional?*"
- Perhaps we should ask, "*What is the integral of the error on the surface or volume?*"

Current Status of Goal-Oriented Mesh Adaptation (J-S. Cagnone)

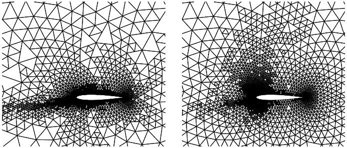


Adaptation based on Lift (left) and Drag (right)

Our Contribution

- Developed a new *p*-adaptive differential-form of the DG Scheme (Based on HT Huynh and ZJ Wang's CPR formulation).
    1. Cagnone, JS and Nadarajah, S, A Stable Interface Element Scheme for the p-Adaptive Lifting Collocation Penalty Formulation, *Journal of Computational Physics, 231*(4), February, 2012.
    2. Cagnone, JS, Vermiere, B, and Nadarajah, S, A p-adaptive LCP formulation for the compressible Navier-Stokes equations, *Journal of Computational Physics* (2012), doi: http://dx.doi.org/10.1016/ j.jcp.2012.08.053.

Current Status of Goal-Oriented Mesh Adaptation (J-S. Cagnone)



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- A new error-norm oriented adjoint-based mesh adaptation.
  1. Cagnone, JS and Nadarajah, S, An error-norm oriented adaptation procedure for the discontinuous Galerkin method

Error-norm adjoint problem (J-S. Cagnone)

Primal flow problem

$$\nabla \cdot \mathcal{F}(\mathbf{u}) = \mathbf{r}(\mathbf{u}) = 0 \quad \text{in } \Omega$$

Cost-function

$$\mathcal{J}(\mathbf{u}) = \frac{1}{2} \int_\Omega (\mathbf{u} - \tilde{\mathbf{u}})^2 \, d\Omega$$

Incorporate PDE constraint into Lagrangian

$$\mathcal{L}(\mathbf{u}, \psi) = \frac{1}{2} \int_\Omega (\mathbf{u} - \tilde{\mathbf{u}})^2 \, d\Omega + \int_\Omega \psi^T \mathbf{r}(\mathbf{u}) \, d\Omega$$

Enforce stationary w.r.t. $\delta \mathbf{u}$

$$\delta \mathcal{L} = \int_\Omega (\mathbf{u} - \tilde{\mathbf{u}}) \, \delta \mathbf{u} \, d\Omega + \int_\Omega \psi^T \mathbf{r}' \delta \mathbf{u} \, d\Omega = 0$$

Error-norm adjoint problem (J-S. Cagnone)

Replace $\mathbf{r}(\mathbf{u}) = \nabla \cdot \mathcal{F}(\mathbf{u})$

$$\delta\mathcal{L} = \int_\Omega (\mathbf{u} - \tilde{\mathbf{u}})\,\delta\mathbf{u}\,d\Omega - \int_\Omega \nabla\psi^T \cdot \mathcal{F}'\delta\mathbf{u}\,d\Omega + \int_{\partial\Omega} \psi^T \mathbf{n} \cdot \mathcal{F}'\delta\mathbf{u}\,ds = 0$$

Achieved by choosing ψ s.t.

$$\begin{cases} (\mathcal{F}')^T \cdot \nabla\psi = \mathbf{u} - \tilde{\mathbf{u}} \& \text{in } \Omega \\ (\mathcal{F}' \cdot \mathbf{n})^T \psi = 0 \& \text{on } \partial\Omega \end{cases}$$

Summary

- Adjoint field equation
- Adjoint boundary condition
- *Connection with L_2 error-norm* $\tilde{\mathbf{u}} \leftarrow \mathbf{u}_h$

Error-norm adjoint problem (J-S. Cagnone)

Taylor expansion of $\mathbf{r}(\mathbf{u})$

$$\mathbf{r}(\tilde{\mathbf{u}}) \approx \cancel{\mathbf{r}(\mathbf{u})} - \mathbf{r}'(\mathbf{u} - \tilde{\mathbf{u}}) \approx -\mathbf{r}'\delta\mathbf{u}.$$

Thus we find

$$\begin{aligned}
\int_\Omega \psi^T \mathbf{r}(\tilde{\mathbf{u}})\, d\Omega &\approx - \int_\Omega \psi^T \mathbf{r}'[\mathbf{u}]\delta\mathbf{u}\, d\Omega \quad \text{(from Taylor exp.)} \\
&= \int_\Omega (\mathbf{u} - \tilde{\mathbf{u}})\delta\mathbf{u}\, d\Omega \quad \text{(from stationarity)} \\
&= \int_\Omega (\mathbf{u} - \tilde{\mathbf{u}})^2\, d\Omega \\
&= \|\mathbf{u} - \tilde{\mathbf{u}}\|_\Omega^2
\end{aligned}$$

Summary

- Inner product is an error-norm estimate
- Adjoint variables quantify sensitivity to residual perturbations
- Element-wise indicator $\eta_k \equiv \int_{\Omega^k} \psi_k^T \mathbf{r}(\mathbf{u}_{h,k})\, d\Omega$

Full algorithm (J-S. Cagnone)

1. Solve flow

$$\mathbf{r}_h(\mathbf{u}_h) = 0$$

2. Solve linear error eqns

$$\mathbf{r}'(\mathbf{u} - \mathbf{u}_h) = -\mathbf{r}(\mathbf{u}_h)$$

3. Solve adjoint

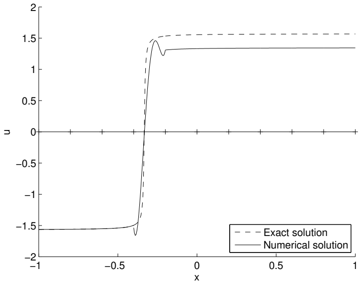
$$\begin{cases} (\mathcal{F}')^T \cdot \nabla\psi = \mathbf{u} - \mathbf{u}_h & \text{in } \Omega \\ (\mathcal{F}' \cdot \mathbf{n})^T \psi = 0 & \text{on } \partial\Omega \end{cases}$$

4. Evaluate error indicator η_k + Adapt mesh

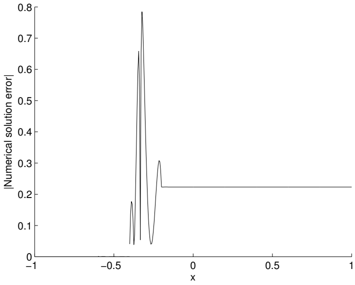
In practice, $\mathbf{r}(\mathbf{u})$ is approximated by a *p*-refined discretization

Steady 1D linear advection (J-S. Cagnone)

$$\frac{\partial u}{\partial x} = f(x) \quad x \in [-1;1]$$

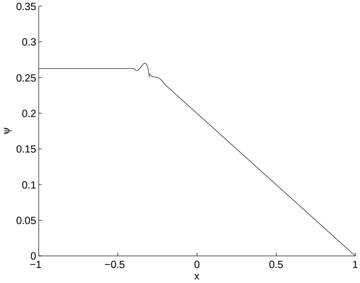


DG-P4 solution

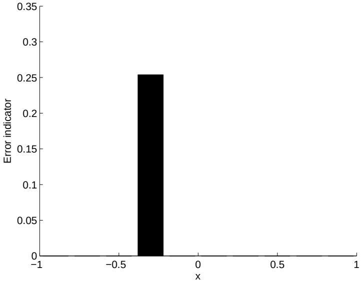


Error profile

Adjoint results (J-S. Cagnone)



Adjoint solution

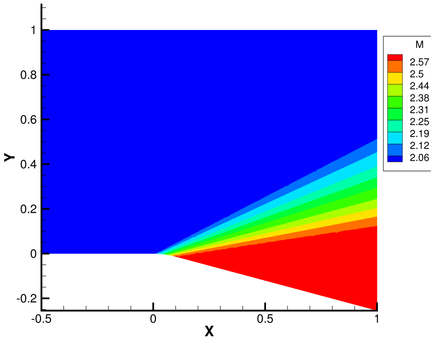


Refinement indicator

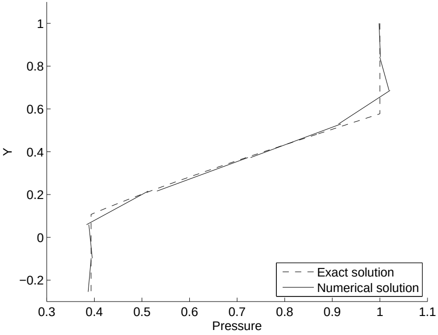
Summary

- Adjoint identifies prime contribution to $\|\mathbf{u} - \mathbf{u}_h\|_\Omega$
- Adequate refinement indicator

$M_\infty = 2$ Prandtl-Meyer expansion fan, $\beta = 15^o$



(a) Mach number

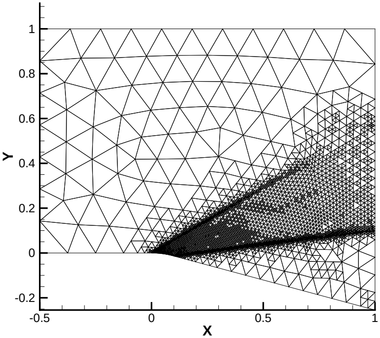


(b) Outflow profile

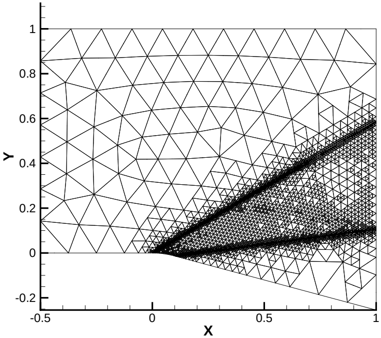
Compare goal/norm-oriented adjoints

- $\mathcal{J}_1 = \int_{\Gamma^+} p(\mathbf{u}) ds$
- $\mathcal{J}_2 = \int_{\Gamma^+} (\mathbf{u} - \mathbf{u}_h)^2 ds$

Adaptively refined meshes

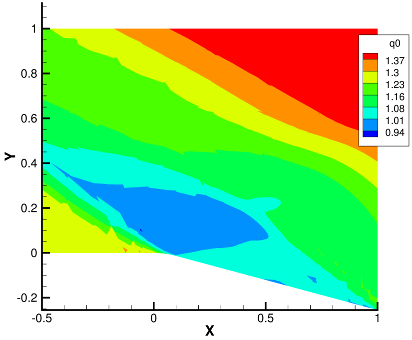


(c) Pressure integral adjoint

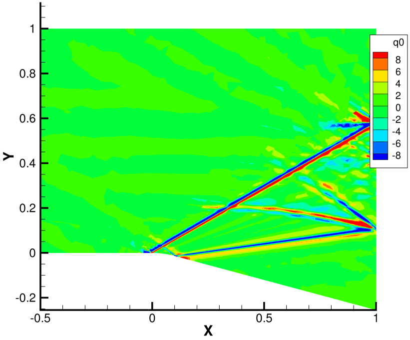


(d) Outflow error-norm adjoint

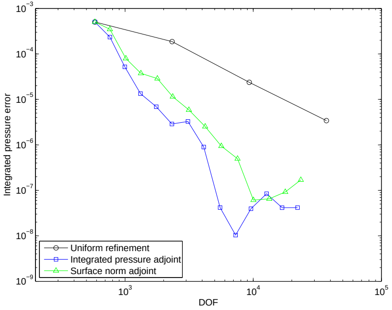
First adjoint component



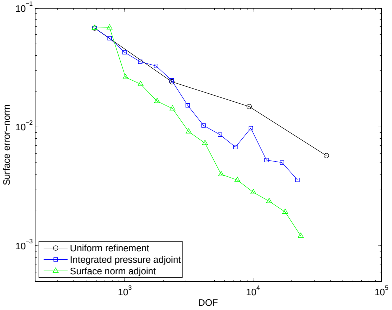
(e) Pressure integral adjoint



(f) Outflow error-norm adjoint



(g) Pressure integral error

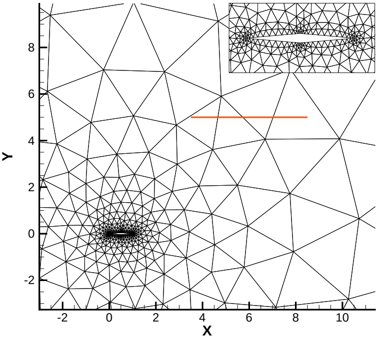


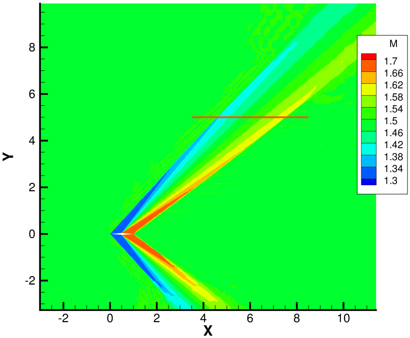
(h) Solution error

Conclusion

- Each adjoint is optimal for its respective cost function
- Error-norm adjoint is useful to minimize actual solution error

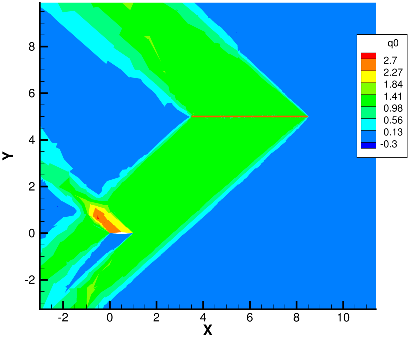
$M_\infty = 1.5$ Supersonic diamond airfoil



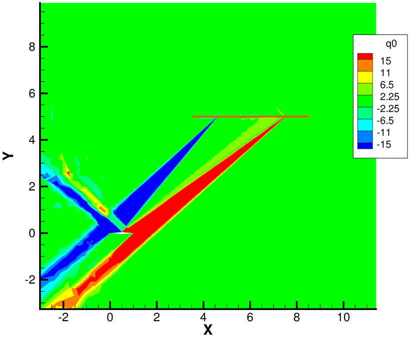


Compare goal/norm-oriented adjoints

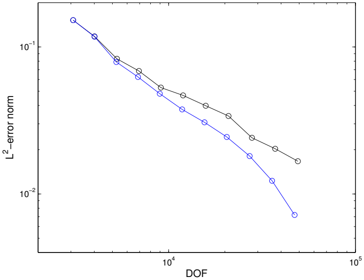
- $\mathcal{J}_1 = \int_S p(\mathbf{u})ds$
- $\mathcal{J}_2 = \int_S (\mathbf{u} - \mathbf{u}_h)^2 d\Omega$



(c) Pressure integral adjoint



(d) Surface norm adjoint



Surface norm error

Summary

- Each adjoint is optimal for its respective cost function
- Norm-oriented predicts more accurate pressure signature

Contributions

- Novel norm-oriented adjoint method
- Verification on supersonic flow problems

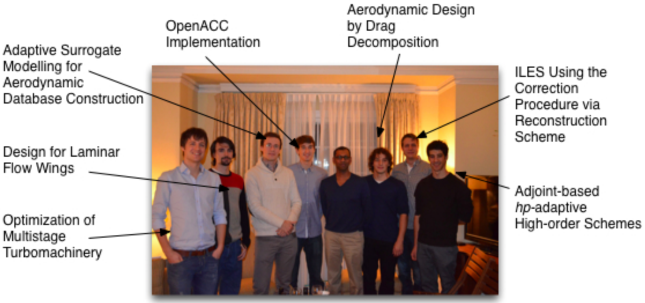
Conclusion

- Adjoint enables identification of error sources
- Correctly accounts for physics & error transport
- Useful of accurate signal capture

Future work

- Algorithmic cost reductions
- *hp*-Adaptation
- Viscous problems

Acknowledgements



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